HOMEWORK (SECTION 9.9)

Theorem (The Remainder Estimation Theorem) *If the function f can be differentiated $n+1$ times in an open interval containing x_0, and if M is an upper bound for $|f^{n+1}(x)|$ in the interval, that is, $|f^{n+1}(x)| \leq M$ for all x in the interval, then*

$$|R_n(x)| \leq \frac{M}{(n+1)!}|x - x_0|^{n+1}$$

for all x in the interval.

Also, please understand the meaning of the remainder $R_n(x)$. In particular, if one uses an nth Taylor polynomial $P_n(x)$ to approximate $f(x)$, then the remainder $R_n(x)$ (i.e. the error) is

$$R_n(x) = f(x) - P_n(x).$$

So, if $R_n \to 0$ as $n \to \infty$, then the Taylor series for f converges to the function $f(x)$.

Please do the following homework problems.

1. Let $f(x) = e^x$.
   (a) Find the Taylor polynomial $P_4(x)$ for $f(x)$ about $x_0 = 1$. Also find the nth Taylor polynomial $P_n(x)$ as well as the Taylor series for $f(x)$ about $x_0 = 1$.
   (b) Use the Reminder Estimation Theorem to find an upper bound for the remainder R_4 when x is in the interval $(-1, 3)$.
2. Let $f(x) = \ln(1 + x)$, $x > -1$.
   (a) Find the Maclaurin polynomial P_3 for $f(x)$. Also find the nth Maclaurin polynomial $P_n(x)$ as well as the Maclaurin series for $f(x)$.
   (b) Use the Reminder Estimation Theorem to find an upper bound for the remainder R_3 when x is in the interval $(-\frac{1}{2}, \frac{1}{2})$.